3RD GRADE

2018-2019 Curriculum Guide

September 10- October 12

<u>Eureka</u>

Module 1: *Properties of Multiplication and Division Solving Problems with Units 2-5 and 10*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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Module 1 Performance Overview

- Students use the language of multiplication as they understand what factors are and differentiate between the size of number and the number of factors in that number. In this module the factors 2, 3, 4, 5, and 10 provide an entry point for moving into more difficult factors in later modules.

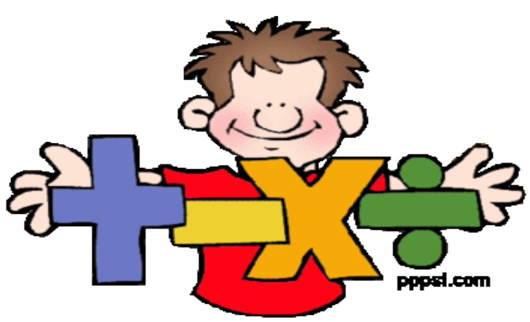
- Topic B extends the study to division. Students understand division as an unknown factor problem, and relate the meaning of unknown factors to either the number or the size of groups. By the end of Topic B students are aware of a fundamental connection between multiplication and division that sets the foundation for the rest of the module

- In Topic C, students use the array model and familiar skip-counting strategies to solidify their understanding of multiplication. They become fluent enough with arithmetic patterns to “add” or “subtract” groups from known products to solve more complex multiplication problems. They apply their skills to word problems using drawings and equations with a symbol to find the unknown factor. This culminates in students using arrays to model the distributive property as they decompose units to multiply.

- In Topic D students model, write and solve division problems with 2 and 3. Consistent skip-counting strategies and the continued use of array models are pathways for students to naturally relate multiplication and division and solidify understanding of their relationship.

- Topic E shifts students from simple understanding to analyzing the relationship between multiplication and division. Practice of both operations is combined—this time using units of 4. Skip-counting, the distributive property, arrays, number bonds and tape diagrams are tools for both operations. Lastly, students learn that the order of numbers in a multiplication problem create equivalent expressions (eg. 3 * 4 = 4 * 3).

- Topic F introduces the factors 5 and 10. In the final lesson of the module, students apply the tools, representations, and concepts they have learned to problem-solving with multi-step word problems using all four operations. They demonstrate the flexibility of their thinking as they assess the reasonableness of their answers for a variety of problem types.



Module 1: *Properties of Multiplication and Division Solving Problems with Units 2-5 and 10*

Pacing: September 10th- October 12th **24 Days**		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Multiplication and the Meaning of the Factors	Lesson 1	Understand *equal groups* of as multiplication. https://www.youtube.com/watch?v
	Lesson 2	Relate multiplication to the array model. https://www.youtube.com/watch?v
	Lesson 3	Interpret the meaning of factors — the size of the group or the number of groups. https://www.youtube.com/watch?v
Topic B: Division as an Unknown Factor Problem	Lesson 4	Understand the meaning of the unknown as the size of the group in division. https://www.youtube.com/watch?v
	Lesson 5	Understand the meaning of the unknown as the number of groups in division. https://www.youtube.com/watch?v
	Lesson 6	Interpret the unknown in division using the array model. https://www.youtube.com/watch?v
Topic C: Multiplication Using Units of 2 and 3	Lesson 7	Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models. https://www.youtube.com/watch?v
	Lesson 8	Demonstrate the commutativity of multiplication and practice related facts by skip-counting objects in array models. https://www.youtube.com/watch?v
	Lesson 9	Find related multiplication facts by adding and subtracting equal groups in array models. https://www.youtube.com/watch?v
	Lesson 10	Model the distributive property with arrays to decompose units as a strategy to multiply. https://www.youtube.com/watch?v
Topic D: Division Using Units of 2 and 3	Lesson 11	Model division as the unknown factor in multiplication using arrays and tape diagrams. https://www.youtube.com/watch?v
	Lesson 12/13	Interpret the quotient as the number of groups or the number of objects in each group using units of 2 and 3. https://www.youtube.com/watch?v https://www.youtube.com/watch?v
Topic E: Multiplication and Division Using Units of	Lesson 14	Skip-Count objects in models to build fluency with multiplication facts using units of 4. https://www.youtube.com/watch?v
	Lesson	Relate arrays to tape diagrams to model the commutative

4	15	property of multiplication. https://www.youtube.com/watch?v
	Lesson 16	Use the distributive property as a strategy to find related multiplication facts. https://www.youtube.com/watch?v
	Lesson 17	Model the relationship between multiplication and division. https://www.youtube.com/watch?v
Topic F: Distributive Property and Problem Solving Using Units of 2–5 and 10	Lesson 18-19	Apply the distributive property to decompose units. https://www.youtube.com/watch?v https://www.youtube.com/watch?v
	Lesson 20	Solve two-step word problems involving multiplication and division and assess the reasonableness of answers. https://www.youtube.com/watch?v
	Lesson 21	Solve two-step word problems involving all four operations and assess the reasonableness of answers. https://www.youtube.com/watch?v
End Of Module Assessment **October 11-12, 2018**		

NJSLS Standards:

3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

- Students develop an initial understanding of multiplication of whole numbers by modeling situations in which there are a specific number of groups with the same number of items in each group.
- Unlike addition, in which each addend represents a certain number of items, in multiplication one factor represents the number of groups and the other factor represents the number of items in each group. The product represents the total number of items in all of the groups.
- Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol '*x*' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.
- To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication, expression (e.g., 5 x 6) students interpret the expression using a multiplication context. They should begin to use the terms, *factor* and *product,* as they describe multiplication.

For example:
Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, 5 x 3 = 15. Describe another situation where there would be 5 groups of 3 or 5 x 3.Sonya earns $7 a week pulling weeds.

After 5 weeks of work, how much has Sonya worked? Write an equation and find the answer.
Describe another situation that would match 7 x 5.

3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

- Division can be understood by thinking in terms of finding a missing factor (either the number of groups or the number of items in a group)
- There are two distinct meanings of division.

Partitive (Fair sharing):
Knowing the total number of items product) and the number of groups (factor) to find the number of items in each group (missing factor).

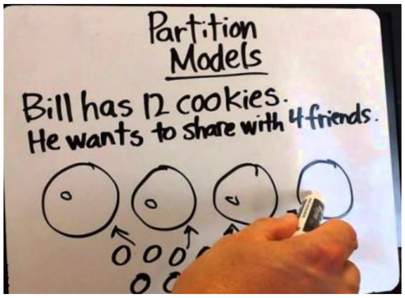
Measurement (Repeated Subtraction):
Knowing the total number of items (product) and the number of items in each group (factor) to find the amount in each group (missing factor)

Type of Division	**Number of Groups**	**Number of Items in group**	**Total number of Items**
Partitive	Known	Unknown	Known
Measurement	Unknown	Known	Known

- Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor).
- To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g., 24 ÷ 6) students interpret the expression in contexts that require both interpretations of division.

For example:
Partition models provide students with a total number and the number of groups. These models focus on the question, "How many objects are in each group so that the groups are equal?"

A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among 4 friends, how many cookies will each friend get?



3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

- Table 2 (below) provides problem sitautions for mulyiplicationa and division. These contexts provide important links to the developing conceptual understaindg of the meaning of multiplication and division.
- Begin with modeling equal group situations and progress to array and area situations. *Comparison situation do not need to be introduced until Grade 4.*
- Students need many opportunities to use concrete materials to model the situations and identify the number of groups and the number of items in a group.
- Once students demonstrate understanding with multiplication situations, use connected division examples in which students identify the total number of objects and explain whether they know the number of groups of the number of items.

3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □ ÷ 3, 6 × 6 = ?.*

- This standard is strongly connected to 3.OA.3 where students solve problems and determine unknowns in equations.
- Students should connect their understanding of modeling and explaining division situations to symbolic notation, writing equations.
- Focusing on the relationship between multiplication and division will help students develop fluency with related fact families.
- Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation.
- Students apply their understanding of the meaning of the equal sign as ”the same as” to interpret an equation with an unknown. When given 4 x ? = 40, they might think:
  - 4 groups of some number is the same as 40
  - 4 times some number is the same as 40
  - I know that 4 groups of 10 is 40 so the unknown number is 10

- The missing factor is 10 because 4 times 10 equals 40.

- Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Examples:

- Solve the equations below:
  24 = ? x 6
  72 ÷ Δ = 9
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = m

3.OA.5	Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

- Properties should not be taught in isolation, but rather should be developed and discussed as part of student experiences. Incorporate opportunities for students to use the properties to develop strategies and patterns to simplify what is happening when they multiply two numbers.

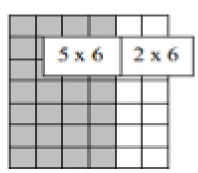
- **Identity Property:** Multiplying a number by 1 does not change the number.
- **Zero Property of Multiplication:** If one of the factors is zero the product is zero.
- **Commutative Property of Multiplication:** Factors represent two different quantities- One factor represents the number of groups and the other factor represents the number of items in each group.

Although 6 x 3 and 3 x 6 have the same product, the actual multiplication situations are not the same.

- **Associative Property of Multiplication:** When multiplying three or more numbers, the product is always the same regardless of their grouping. This property is helpful in developing strategies for mental computation and decomposing factors to help students learn more difficult multiplication facts.
  Solving for the total number of items (the product) in “a groups with b items” (a x b) and c groups of these groups is the same as thinking about “a” groups of (b x c). Both ways of putting the groups of items together result in the same product because, regardless of how the groups are put together, the same number of items are being combined.

- **Distributive property of Multiplication:** Explored in the context of composing and decomposing factors. This will help students learn more difficulty basic facts.

For example, in the picture below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.



3.OA.6	Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

- Multiplication and division are inverse operations and that understanding can be used to find the unknown.
- Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Example:
3 x 5 = 15 & 5 x 3 = 15
15 ÷ 3 = 5 & 15 ÷ 5 = 3

- Students understand that multiplication and division are inverse operations and that understanding can be used to find the unknown.
- Number Bonds demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Examples:
5 x 9 = 45 & 9 x 5 = 45
45 ÷ 5= 9 & 45 ÷ 9 = 5

Equations in the form of a ÷ b =c and c = a ÷ b need to be used interchangeably, with the unknown in different positions.

3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

- This is a culminating standard to show the outcome of multiplication and division understanding in this domain and fluency within 100.

- By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts.

- Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

Strategies students may use to attain fluency include:

• Multiplication by zero and one
• Doubles (2s facts), Doubling twice (4s), Doubling three times (8s)
• Tens facts (relating to place value, 5 x 10 is 5 tens or 50)
• Five facts (half of tens)
• Skip counting (counting groups of __ and knowing how many groups have been counted)
• Square numbers (ex: 3 x 3)
• Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3)
• Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6)
• Turn-around facts (Commutative Property)
• Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24)
• Missing factors

General Note: ***Students should have exposure to multiplication and division problems presented in both vertical and horizontal written forms.***

3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

- Students solve two-step problems that include more than one operation by representing the information using concrete models, pictures including bar models, and number lines. Writing equations begins with making connections between the representations and the symbolic notation (equations).

- Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use that make most sense them.

Determining whether answers are reasonable by using number sense, understanding the context, the meaning of operations using mental computation strategies, and estimation strategies cannot be overemphasized as students work with all of their ideas embedded in this Standard.

- Using a letter standing for the unknown quantity should explicitly connect to previous work with identifying missing information that was represented by a box, underscore, or other symbols.

- When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.

Example:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did they travel?

Student 1:	**Student 2:**	**Student 3:**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundred that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30. I know my answer be about 500.

Major Clusters Supporting Additional Clusters

Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 1 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Diagnostic Assessment (IREADY)		1-2 blocks	Individual
Eureka Math Module 1: Properties of Multiplication and Division and Solving Problems with units of 2-5 and 10			
Authentic Assessment #1	3.OA.1-8	30 mins	Individual
Optional Mid Module Assessment	3.OA.1-8	1 Block	Individual
Optional End of Module Assessment	3.OA.1-8	1 Block	Individual

Third Grade Ideal Math Block

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Whole
Exit Ticket: Independent

50-60 min.

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

20- 30 min.

M: Meet with the teacher	A: Application/ Problem Solving	T: Technology	H: Hands on Activities

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements			
CCSS	**Evidence Statement**	**Clarification**	**MP**
3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.	• Tasks involve interpreting rather than calculating products in terms of equal groups, arrays, area, and/or measurement quantities. (See CCSSM, Table 2, Common multiplication and division situations, p. 89.) For example, "the total number of books if 5 shelves each have 7 books" can be represented by the expression 5x7 rather than "Marcie placed 7 books on each of 5 shelves. How many books does she have?" • Tasks do not require students to interpret products in terms of repeated addition, skip counting, or jumps on the number line. • The italicized example refers to describing a real-world context, but describing a context is not the only way to meet the standard. For example, another way to meet the standard would be to identify contexts in which a total can be expressed as a specified product.	MP 2,4
3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.	• Tasks involve interpreting rather than calculating quotients in terms of equal groups, arrays, area, and/or measurement quantities. (See CCSSM, Table 2, Common multiplication and division situations, p. 89.). For example, "35 books are placed equally on 7 shelves" can be represented by the expression 35 ÷ 7 rather than "Marcie has 35 books. She placed the same number on each of 7 shelves. How many books did she place on each shelf?" • Tasks do not require students to interpret quotients in terms of repeated subtraction, skipcounting, or jumps on the number line. • The italicized example refers to describing a real-world context, but describing a context is not the only way to meet the standard. For example, another way to meet the standard would be to identify contexts in which a number of objects can be expressed as a specified quotient. • Half the tasks require interpreting quo-	MP 2,4

		tients as a number of objects in each share and half require interpreting quotients as a number of equal shares.	
3.OA.3-1	Use multiplication within 100 (both factors less than or equal to 10) to solve word problems in situations involving equal groups, arrays, or area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	• All products come from the harder three quadrants of the times table (a × b where a > 5 and/or b > 5). • 75% of tasks involve multiplying to find the total number (equal groups, arrays); 25% involve multiplying to find the area. • For more information see CCSS Table 2, Common multiplication and division situations, p. 89 and the OA Progression.	MP 1,4
3.OA.3-2	Use multiplication within 100 (both factors less than or equal to 10) to solve word problems in situations involving measurement quantities other than area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	• All products come from the harder three quadrants of the times table (a × b where a > 5 and/or b > 5). • Tasks involve multiplying to find a total measure (other than area). • For more information see CCSS Table 2, Common multiplication and division situations, p. 89 and the OA Progression.	MP 1,4
3.OA.3-3	Use division within 100 (quotients related to products having both factors less than or equal to 10) to solve word problems in situations involving equal groups, arrays, or area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	• All quotients are related to products from the harder three quadrants of the times table (a × b where a > 5 and/or b > 5). • Tasks using this Evidence Statement will be created equally among the following: • dividing to find the number in each equal group or in each equal row/column of an array; • dividing to find the number of equal groups or the number of equal rows/columns of an array; and • dividing an area by a side length to find an unknown side length. • For more information see CCSS Table 2, Common multiplication and division situations p. 89 and the OA Progression.	MP 1,4
3.OA.3-4	Use division within 100 (quotients related to products having both factors less than or equal to 10) to solve word problems in situations involving measurement quantities other than area, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem	• All quotients are related to products from the harder three quadrants of the times table (a × b where a > 5 and/or b > 5). • Half the tasks involve finding the number of equal pieces and half involve finding the measure of each piece. • For more information see CCSS Table 2, Common multiplication and division	MP 1,4

		situations, p. 89 and the OA Progression.	
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = � ÷ 3, 6 × 6 = ?.	• Tasks do not have a context. • Only the answer is required. • All products and related quotients are from the harder three quadrants of the times table (a × b where a > 5 and/or b > 5).	
3.OA.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8	• All products and related quotients are from the harder three quadrants of the times table (a × b where a > 5 and/or b > 5).	
3.OA.7-1	Fluently multiply and divide within 25. By end of grade 3, know from memory all products of two one-digit numbers.	• Tasks do not have a context. • Only the answer is required. • Tasks require finding products and related quotients accurately. For example, each 1-point task might require four or more computations, two or more multiplication and two or more division. • Tasks are not timed.	
3.OA.7-2	Fluently multiply and divide within 100. By the end of Grade 3, know from memory all products of two one-digit numbers.	• Tasks do not have a context. • Only the answer is required. • Tasks require finding of products and related quotients accurately. For example, each 1- point task might require four or more computations, two or more multiplication and two or more division. • 75% of tasks are from the harder three quadrants of the times table (a × b where a > 5 and/or b > 5). • Tasks are not timed.	
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	• Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation. • Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown. • Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situ-	MP 1,4

		ation types with unknowns in various positions (see CCSSM, Table 1,Common addition and subtraction situations, p. 88; CCSSM, Table 2, Common multiplication and division situations, p. 89; and the document for the OA Progression).	

Number Talks

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

The teacher asks questions:

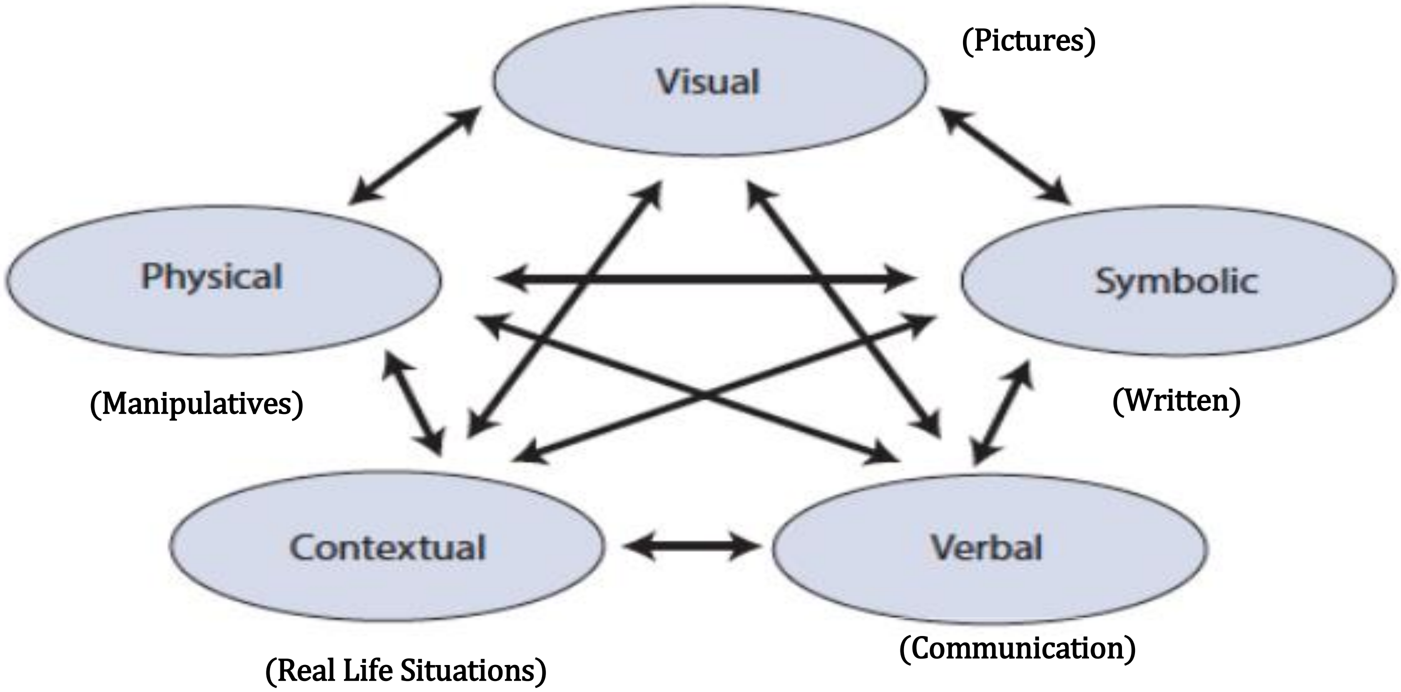
- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?

- What was the first thing your eyes saw, or your brain did?

Student Name: ________________________________ Task: ______________________ School: ________ Teacher: __________ Date: ________

"I CAN….."	STUDENT FRIENDLY RUBRIC				SCORE
	...a start 1	...getting there 2	...that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: "Doing Stage": Physical manipulation of objects to solve math problems.

Pictorial: "Seeing Stage": Use of imaged to represent objects when solving math problems.

Abstract: "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

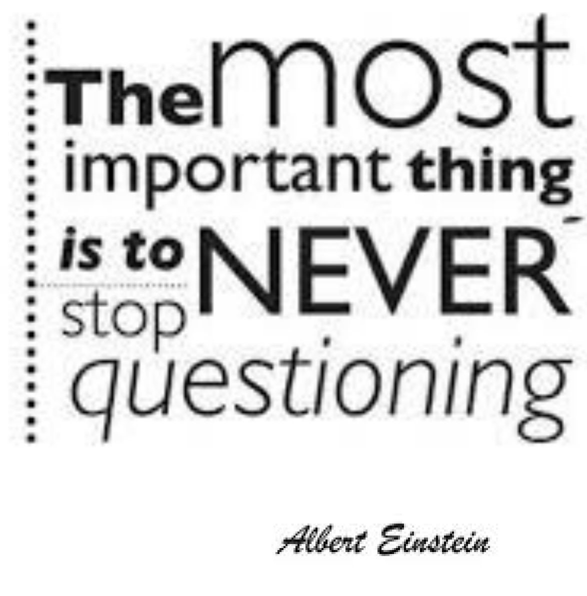
Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts.** What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

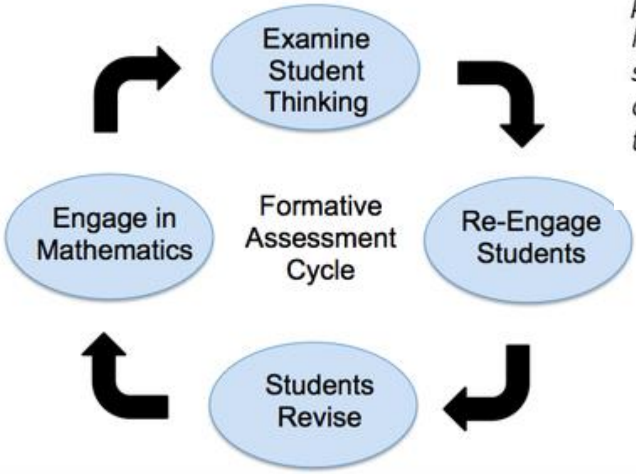
- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)



Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1	**Make sense of problems and persevere in solving them** In **third** grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** In **third** grade, students should recognize that number represents a specific quantity. They connect quantity to written symbols and create logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities
3	**Construct viable arguments and critique the reasoning of others** In **third** grade, mathematically proficient students may construct viable arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" and "Why is it true?" They explain their thinking to others and respond to others' thinking.
4	**Model with mathematics** Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language) drawing pictures, using objects, acting out, making chart, list, or graph, creating equations etc…Students need opportunities to connect different representations and explain the connections. They should be able to use all of the representations as needed. **Third** graders should evaluate their results in the context of the situation and reflect whether the results make any sense.
5	**Use appropriate tools strategically** **Third** graders should consider all the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For example, they might use graph paper to find all possible rectangles with the given perimeter. They compile all possibilities

	into an organized list or a table, and determine whether they all have the possible rectangles.
6	**Attend to precision**
	Mathematical proficient **third** graders develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying their units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle the record their answer in square units.
7	**Look for and make use of structure**
	In **third** grade, students should look closely to discover a pattern of structure. For example, students properties of operations as strategies to multiply and divide. (commutative and distributive properties.
8	**Look for and express regularity in repeated reasoning**
	Mathematically proficient students in third **grade** should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7x8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ____ Time: ___________

NJSLS.:

____

____

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE

Week of: ________________

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

Third Grade PLD Rubric

Got It		Not There Yet		
Evidence shows that the student essentially has the target concept or big math idea.		Student shows evidence of a major misunderstanding, incorrect concepts or procedure, or a failure to engage in the task.		
PLD Level 5: 100% Distinguished command	**PLD Level 4: 89% Strong Command**	**PLD Level 3: 79% Moderate Command**	**PLD Level 2: 69% Partial Command**	**PLD Level 1: 59% Little Command**
Student work shows **distinguished levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes an **efficient and logical progression** of mathematical reasoning and understanding.	Student work shows **strong levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes a **logical progression** of mathematical reasoning and understanding.	Student work shows **moderate levels of understanding** of the mathematics. Student **constructs** and **communicates** a **complete response** based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes a **logical but incomplete progression** of mathematical reasoning and understanding. Contains **minor errors.**	Student work shows **partial understanding** of the mathematics. Student **constructs** and **communicates** an **incomplete response** based on student's attempts of explanations/ reasoning using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes an **incomplete or illogical progression** of mathematical reasoning and understanding.	Student work shows **little understanding** of the mathematics. Student **attempts** to **constructs** and **communicates** a response using the: • properties of operations • relationship between addition and subtraction relationship • Use of math vocabulary Response includes **limited evidence of the progression** of mathematical reasoning and understanding.
5 points	**4 points**	**3 points**	**2 points**	**1 point**

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

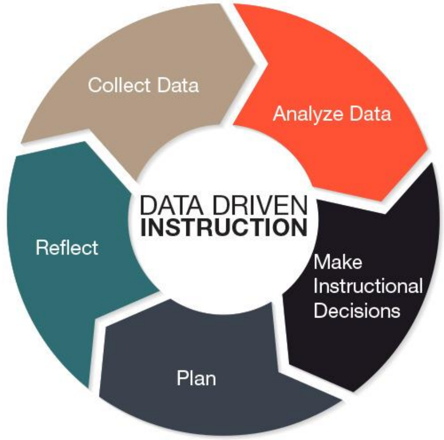
Answering these questions will help inform instructional decisions and will influence lesson planning.

Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?



Now it is time to begin the analysis again.

Data Analysis Form School: ________________ Teacher: ________________________ Date: _____________

Assessment: ____ NJSLS: ____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

Cookie Dough

Name:______________________________________

Clear Creek School is fundraising.
They are selling Cookie Dough in tubs.

Chocolate Chip Cookie Dough	Peanut Butter Cookie Dough	Oatmeal Cookie Dough
$5 a tub	$4 a tub	$3 a tub

1. Jill sold 2 tubs of Oatmeal Cookie Dough. How much money did she raise?

2. Joe sold 4 tubs of Peanut Butter Cookie Dough and 4 tubs of Chocolate Chip Cookie Dough. How much money did her raise in all? Show how you figured it out.

3. Jade sold only Peanut Butter Cookie Dough and she raised $32. How many tubs did she sell? Show how you figured it out.

4. Jermaine's mother loves oatmeal cookies. She has $20 to spend. What is the greatest number of tubs of Oatmeal Cookie Dough she can buy? Explain how you figured this out.

Authentic Assessment #7 Scoring Rubric – Cookie Dough

3.OA.7: Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
Mathematical Practice:1, 6
Type: Individual

<table>
<tr><td colspan="5">SOLUTION:
1. $6
2. $36
3. 8 tubs
4. 6 tubs</td></tr>
<tr><td>Level 5:
4 Correct Answers
Distinguished Command</td><td>Level 4:
3 Correct Answers
Strong Command</td><td>Level 3:
2 Correct Answers
Moderate Command</td><td>Level 2:
1 Correct Answers
Partial Command</td><td>Level 1:
No Correct Answers
No Command</td></tr>
<tr><td>Clearly constructs and communicates a complete response based on explanations/reasoning using the:
• Properties of operations
• Relationship between multiplication and division

Response includes an <u>efficient</u> and logical progression of steps.</td><td>Clearly constructs and communicates a complete response based on explanations/
reasoning using the:
• Properties based on place value
• properties of operations
• relationship between addition and subtraction

Response includes a <u>logical</u> progression of steps.</td><td>Constructs and communicates a complete response based on explanations/
reasoning using the:
• Properties of operations
• Relationship between multiplication and division

Response includes a <u>logical but incomplete</u> progression of steps. Minor calculation errors.</td><td>Constructs and communicates anincomplete response based on explanations/
reasoning using the:
• Properties of operations
• relationship between multiplication and division

Response includes an <u>incomplete or Illogical</u> progression of steps.</td><td>The student shows no work or justification</td></tr>
</table>

Resources

Great Minds
https://greatminds.org/

Embarc
https://embarc.online/

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center:
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.
CRP2. Apply appropriate academic and technical skills.
CRP3. Attend to personal health and financial well-being.
CRP4. Communicate clearly and effectively and with reason.
CRP5. Consider the environmental, social and economic impacts of decisions.
CRP6. Demonstrate creativity and innovation.
CRP7. Employ valid and reliable research strategies.
CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
CRP9. Model integrity, ethical leadership and effective management.
CRP10. Plan education and career paths aligned to personal goals.
CRP11. Use technology to enhance productivity.
CRP12. Work productively in teams while using cultural global competence.

For additional details see **21st Century Career Ready Practices** .

References

"Eureka Math" *Great Minds.* 2018 < https://greatminds.org/account/products>